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DYNAMIC STRUCTURE OF THE LOWER ATMOSPHERE

By

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and

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August 1968

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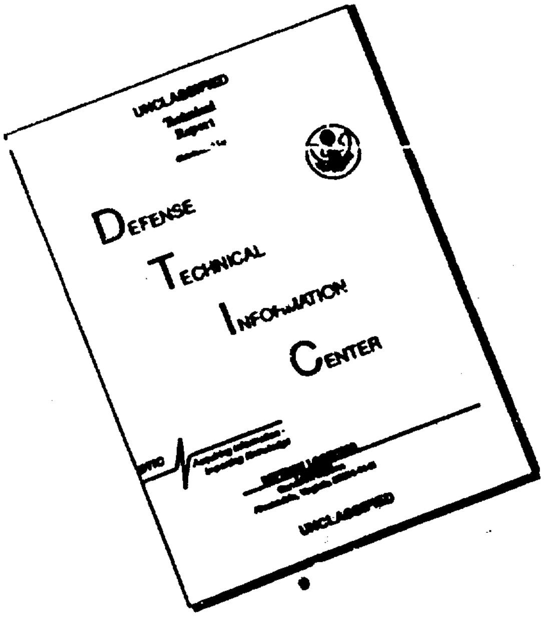

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DYNAMIC STRUCTURE OF THE LOWER ATMOSPHERE

I. Introduction

The work carried out under grant DA-AMC-28-043-67-G6 can be divided as follows:

1. Continuation of a theoretical study of vortex flow and its stability. This work had been started under grant DA-AMC-36-039-63-G-1 by K. H. Bergman. Theoretical analysis and preliminary results of this investigation were published in the final report of this grant (1967). Bergman will continue this work during the summer of 1968 and hopefully finish a Ph.D. thesis.
2. Field experiments comparing the sonic anemometer with other instruments measuring the vertical heat flux and the momentum flux. The results of a field experiment in Hay, N.S.W., Australia during May 1966 have been reported by Businger, Miyake, Dyer and Bradley (1967). In this report some preliminary results of the Davis field experiment which was carried out in April and May of 1967. The reduction and analysis of data obtained during this experiment will continue throughout the summer and fall of 1968.
3. A study of the statistical spatial distribution of buoyant plumes has been started. Mr. A. S. Frisch gives a summary of his proposed study.

Reprints of papers that have appeared during the past year have been attached at the end of the report.

II. The Davis Field Experiment

by

J. A. Businger

The main purpose of the Davis field experiment was the comparison of various instruments and techniques designed to obtain the turbulent fluxes in the atmospheric surface layer. The University of Washington participated with the following instruments:

a. one 3-D sonic anemometer (Kaijo Denki)

b. one 1-D sonic anemometer

c. DISA hot wire anemometer

d. Cup anemometer (Beckman Whitley)

e. Thermocouple dry

f. Thermocouple wet

g. Heat flux bar

i. Propeller anemometer (Gill)

The data obtained with these instruments were multiplexed, using 12 I.R.I.G. channels, on a 1/4" high fidelity tape recorder. All the information was collected in analog form to be reduced with an analog data reduction system at the University of Washington. During the first week seven channels were also directly recorded on an instrumentation tape recorder (Ampex F.R. 1300).

Besides from our own sensors, analog signals of some other instruments were recorded during a number of runs notably the anemoclinometer from the University of Wisconsin and a humidity sensor (BaF) from N.B.S., Washington, D.C.

A summary of the recorded data is given in Table I and II. Table I indicates

the period that observations were taken by the University of Washington group. Table II summarizes what information was obtained.

Upon inspection of all the analog signals it turned out that only a relatively small number of runs was complete without any instrument failure. These runs have been used for initial reduction and analysis and are marked with an asterisk in Table I. Some of the results related to the heat flux are given in Table III.

Although the data reduction has not been completed several studies analyzing the data are underway or completed.

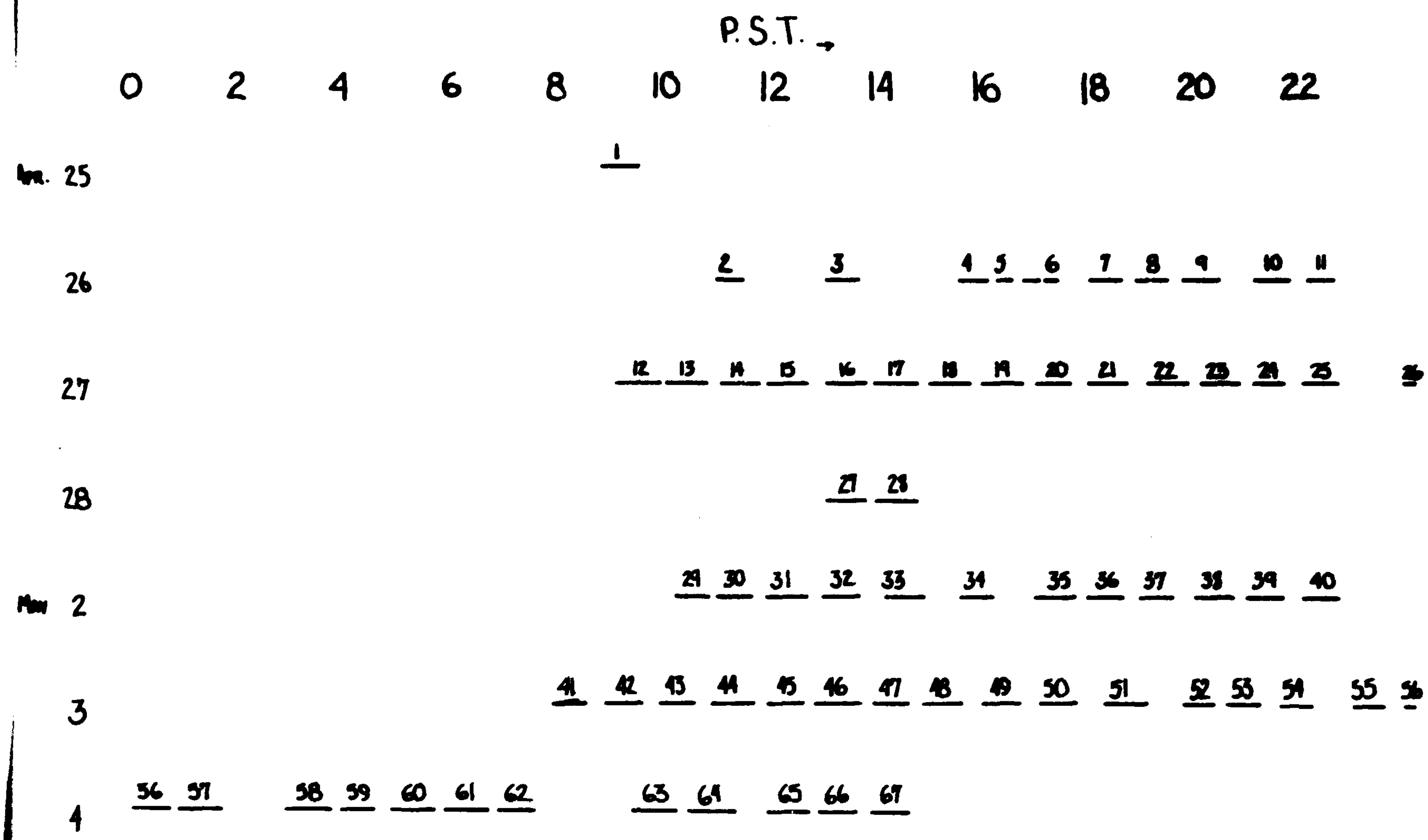
a. It has been shown experimentally that the third moment term in the expression for the heat flux is negligible using data from Davis and Hay, N.S.W. Australia (Businger and Miyake, 1968).

b. A study has been made by K. Sahashi of the direct determination of the water vapor flux using the sonic anemometer and a wet bulb thermocouple. This study will soon appear as a Ph.D. thesis from the University of Kyoto, Japan.

c. Comparison of the performance of the sonic anemometer and of the anemoclinometer is being carried out. The agreement so far is not as good as expected when the data were monitored.

d. Analysis of measurements with the heat flux bar shows that even with extensive precautions of shielding the radiation error is too large to allow accurate heat flux determination.

Several more specific studies will be carried out with the Davis data, including analysis of high frequency observations with the DISA anemometer and of the performance of the Gill propeller anemometer.



TIME SERIES OF OBSERVATION
AT DAVIS CALIFORNIA 1967

TABLE I

TABLE IIa

Summary of Measured Parameters

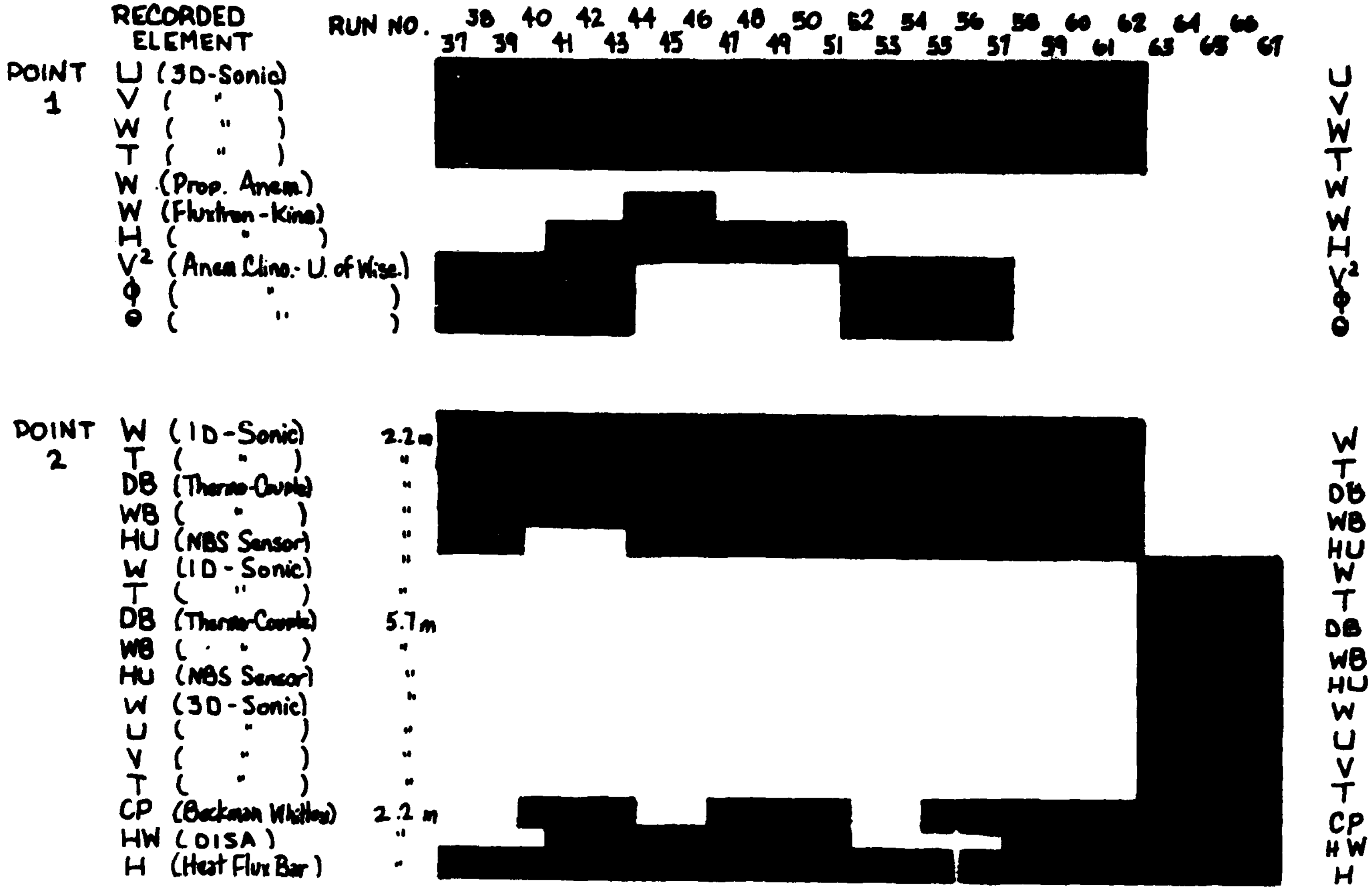
ABBREVIATION IN "RECORDED ELEMENT"

U	Horizontal component of wind velocity
V	"
W	Vertical component of wind velocity
T	Air temperature
H	Heat flux
V	Pressure by horizontal wind
ϕ	Elevation angle of wind velocity
θ	Azimuth angle of wind velocity
DB	Dry-bulb temperature
WB	Wet-bulb temperature
HU	Relative humidity by NBS sensor
CP	Wind speed by cup anemometer
HW	Wind speed by hot-wire anemometer

OBSERVATION POINT

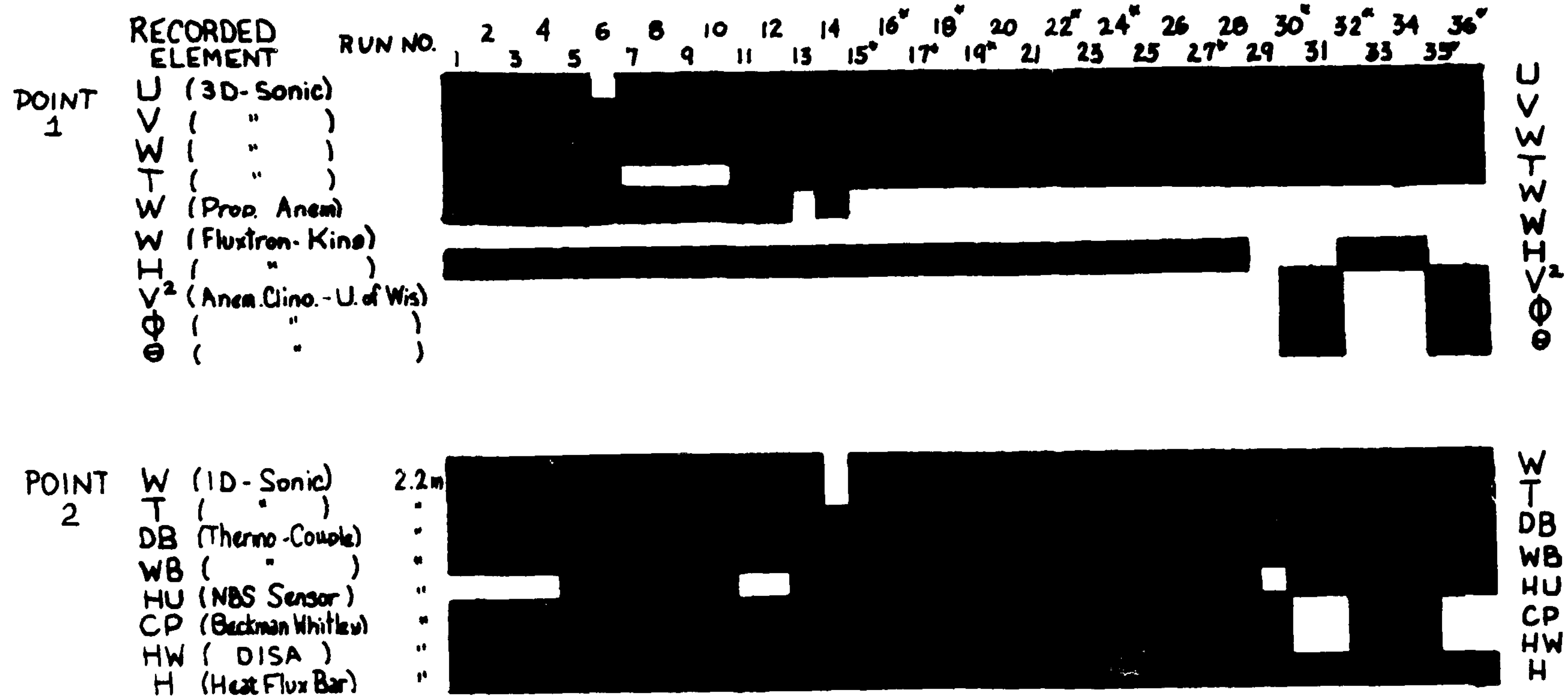
Point 1:	Near the center of the site. ca. 200 m apart from the recording van.
Point 2:	ca. 50 m apart from the recording van.

TABLE IIb



RECORDED ELEMENT
RUN NO.
37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67
POINT 1
U (3D-Sonic)
V (")
W (")
T (")
W (Prop. Anem.)
W (Fluxtron-Kine)
H (")
V² (Anem Clino.- U. of Wisc.)
φ (")
θ (")
POINT 2
W (1D-Sonic) 2.2 m
T (") "
DB (Thermo-Couple) "
WB (") "
HU (NBS Sensor) "
W (1D - Sonic) "
T (") "
DB (Thermo-Couple) 5.7 m
WB (") "
HU (NBS Sensor) "
W (3D - Sonic) "
U (") "
V (") "
T (") "
CP (Beckman Whitley) 2.2 m
HW (DISA) "
H (Heat Flux Bar) "
U V W T W W H V² φ θ
W T DB WB HU W T DB WB HU W U V T CP HW H

TABLE IIc



SHADING SHOWS THE DATA ACQUIRED

* SEE TEXT

TABLE III

Variances and Covariances of a Selected Number of Runs

DATE	RUN	STARTING TIME	$\overline{T_1^2}$ °C²	$\overline{w_1^2}$ m^2sec^{-2}	$\overline{w_1T_1}$ m°C sec^{-1}	$\overline{T^2}$ °C²	$\overline{w_1T}$ m°C sec^{-1}	$\overline{w_3^2}$ m^2sec^{-2}	$\overline{T_3^2}$ °C²	$\overline{w_3T_3}$ m°C sec^{-1}
April 27	15	1155	0.65	0.111	-	0.578	-	0.346	0.53	0.143
	16	1300	0.54	0.150	0.109	0.516	0.097	0.365	0.50	0.185
	17	1355	0.69	0.163	0.131	0.312	0.083	0.414	0.392	0.145
	18	1459	0.175	0.125	0.063	0.21	0.062	0.355	0.23	0.105
	19	1555	0.136	0.130	0.024	0.133	0.012	0.365	0.102	0.024
	22	1856	0.132	0.020	-0.017	0.187	-0.019	0.065	0.098	-0.027
	24	2100	0.138	0.005	-0.002	0.256	-0.003	0.008	0.079	-0.005
April 28	27	1256	0.76	0.058	0.083	1.35	0.120	0.166	0.52	0.141
May 2	30	1104	0.52	0.051	0.097	-	-	0.022	0.51	0.054
	32	1402	0.54	0.055	0.106	-	-	0.087	0.355	0.093
	35	1748	0.71	0.090	-0.016	-	-	0.035	0.089	-0.008
	36	1852	0.305	0.004	-0.026	-	-	0.011	0.27	-0.017

III. A Statistical Analysis of Convective Elements In the Unstable Boundary Layer

by

A. S. Frisch

The purpose of this study of the lower atmosphere in the unstable state is to describe some of the characteristics of convective elements and the surrounding environment in varying states of instability. These convective elements appear to be more or less organized structures which are superimposed upon the random turbulent flow. They have been described by various investigators such as Taylor (1958) and Priestley (1959), as asymmetrical fluctuations of temperature with a duration of ten to twenty seconds when observations are made from a fixed site and they show a strong coherence with height. In other studies (Vulf'son, 1964; Warner and Telford, 1967) measuring temperature from aircraft, the assumption was made that the horizontal cross sections of these elements were circular, however not enough observations were made of the temperature to draw any quantitative conclusion. This study will involve simultaneous measurements from aircraft and a tower located in the vicinity of the aircraft flight pattern. The aircraft will be used to statistically determine the temperature distribution and from this the heat flux and vertical velocity field of a horizontal cross section of these convective features. The use of the aircraft is required for observing a large number of the convective elements in order to assure an adequate estimate of the temperature structure in a relatively short interval of time.

The temperature will be measured in two directions by the aircraft, one along the direction of the wind, the other in a transverse direction. These

data will be analyzed using a generalization of a statistical modeling method developed by N. I. Vul'fson in order to describe some of the statistical features of the field of convective elements. (See appendix.)

Since the data are being observed in two directions, a comparison can be made between these observations to test the validity of the model. In addition, the model chosen is one that seems reasonable, but others can also be used, although, if it is too complicated it may require much more refined techniques of measurement than are available. From the parameters obtained by use of the model, inferences can be made about the number and the size distribution of the convective elements in a horizontal plane at different degrees of stability and heights.

In addition to aircraft measurements, a tower will be equipped with instruments to measure temperature and wind speed profiles; as well as instantaneous vertical velocity and temperatures at some of the heights the aircraft measurements are made. All these measurements will be taken simultaneously with that of the aircraft. These tower measurements will be used to compute several quantities of the convective elements and the surrounding environment. Some of the quantities that can be examined are $\overline{w'^2}$, $\overline{w'^3}$, $\bar{\theta}\bar{w}$, $\bar{w}$, $\overline{\theta' w'}$, internal and external to the field of convective elements.

The measurements from the tower can be used in conjunction with aircraft measurements to determine some of the dynamics of the convective elements. For example, since w and θ are measured at different heights on the tower, w can be averaged at each height during the time that the temperature is in the interval from θ_n to $\theta_n + \Delta\theta$. Since each band of temperature θ_n to $\theta_n + \Delta\theta$ corresponds to a two dimensional ring with axis a_n and b_n (determined by the aircraft) the dis-

tribution of θ and w can be found this way for an average size convective element. From continuity, the average radial velocity can be found through a vertical section defined between the sensors. If the boundary layer is extremely unstable, then it may be possible to define a convective element in cylindrical co-ordinates and compare the terms measured in the heat flux and energy equation with those computed from various theoretical models that have been developed for this case.

Field Measurements

A field trip was made to the Hanford reservation to estimate some of the statistical quantities of the convective elements before a more elaborately instrumented field trip is to be made. Support was provided by the Battelle Northwest Corporation. There were essentially three kinds of data being recorded simultaneously, i.e., temperature and vertical velocity measured from the 400' Hanford meteorological tower, velocity and temperature profiles from a 32m portable mast, and horizontal temperature measurements obtained from an aircraft flying at different altitudes.

Heat Flux: The heat flux can be computed from the measurements made at several heights during the series of observations. There were three instruments used to determine the heat flux, they were: (1) Fluxatron, (2) Sonic Anemometer-Thermometer, (3) Propeller Anemometer and Thermocouple. During the first part of the experiment, there was a comparison with the fluxatron and the sonic anemometer at a 22' height.

The propeller anemometer and thermocouple were mounted at 250' for most of the measurements. The sonic anemometer-thermometer was later mounted at 100'

during the last day of the measurements.

Profiles: Since it is desirable to relate the convective features to easier measured quantities, temperature and wind speeds were recorded at different heights on the large 400' tower and also from a smaller portable mast. From these data, $\frac{\partial \bar{T}}{\partial z}$ and $\frac{\partial \bar{v}}{\partial z}$ and related quantities will be computed.

Aircraft temperature measurements: Temperature was measured by a thermocouple mounted on a strut of an L-19 aircraft. The thermocouple signal was amplified and converted to an FM signal and recorded on a Sony Model 800 tape recorder. The direction of the flight was governed by the prevailing wind direction and the amount of unobstructed path for the aircraft to fly over. The paths taken by the aircraft are shown in Figure 1 .

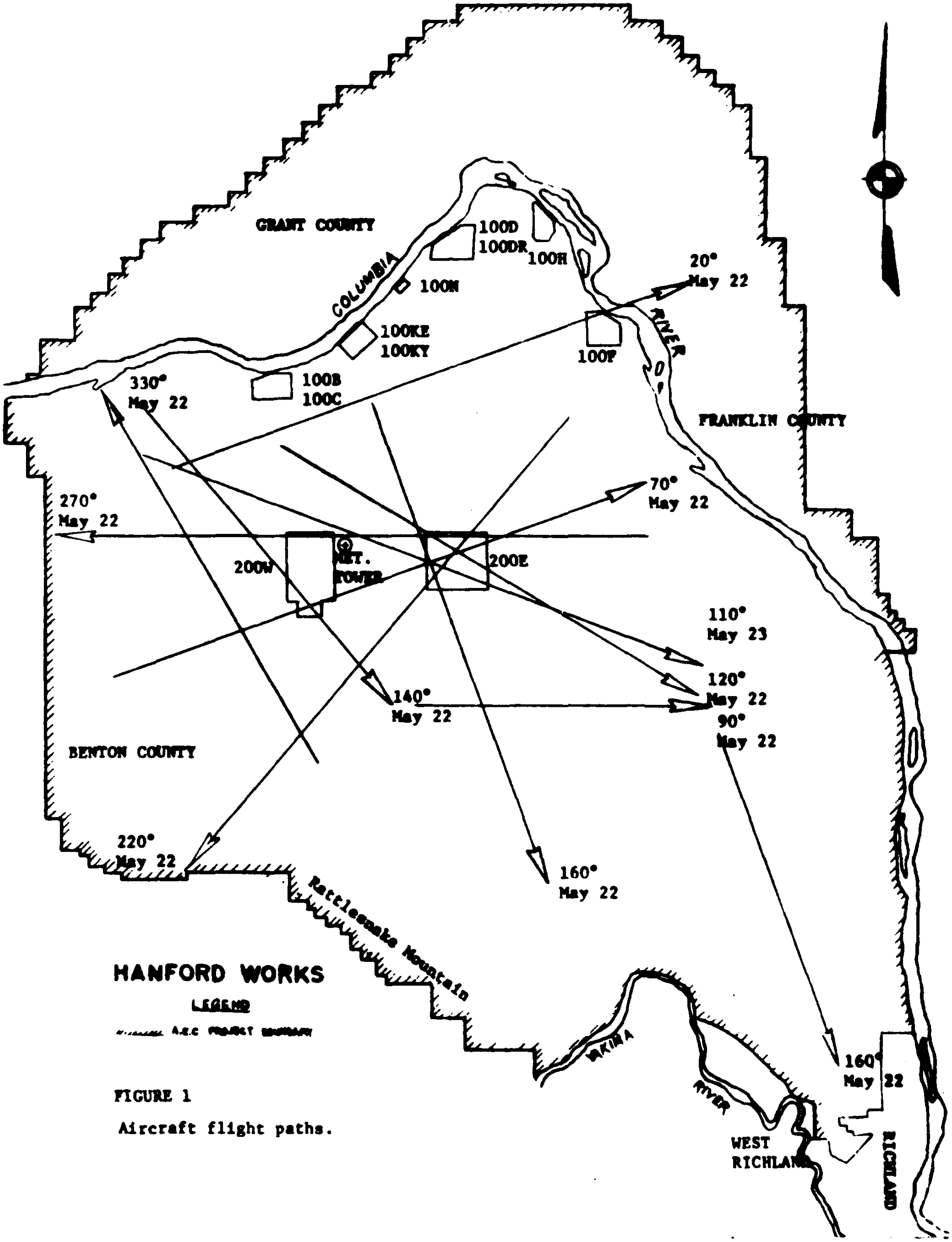


FIGURE 1

Aircraft flight paths.

APPENDIX

Method

Laboratory experiments indicate that convective elements are approximately circular or elliptical in cross section. In the following analysis it is assumed that the elements can be approximated by ellipses because then it can be shown that there are certain relationships between the moments of the intersected lengths of the elements in the two directions. A convective element will be defined as an element having a temperature greater than the average temperature, θ_r. The element will then be subdivided into temperature increments $\theta_n \equiv \theta_r + n\Delta\theta$ where $n = 0,1,2,3.....$, and $\Delta\theta$ is a temperature increment.

The aircraft will intersect these elements at random for intervals of time that depend on the distance from the axis of the element, the size of the element, and the speed of the airplane. These intervals of time can be converted into lengths by multiplying the time intervals by the air speed of the aircraft.

It will be assumed that one of the two axis of each element will be parallel to the wind direction.

Let the axis of the ellipse that lies in the x direction be represented by b, and that in the y direction by a (See Fig. 2). Furthermore assume a similarity between a and b given by:

$$a = \frac{m_n b}{1 - \alpha_n m_n b} \tag{1}$$

$$b = \frac{a}{m_n(1 + \alpha_n a)} \tag{2}$$

If the plumes are circles, then $\alpha_n = 0$; $m_n = 1$.

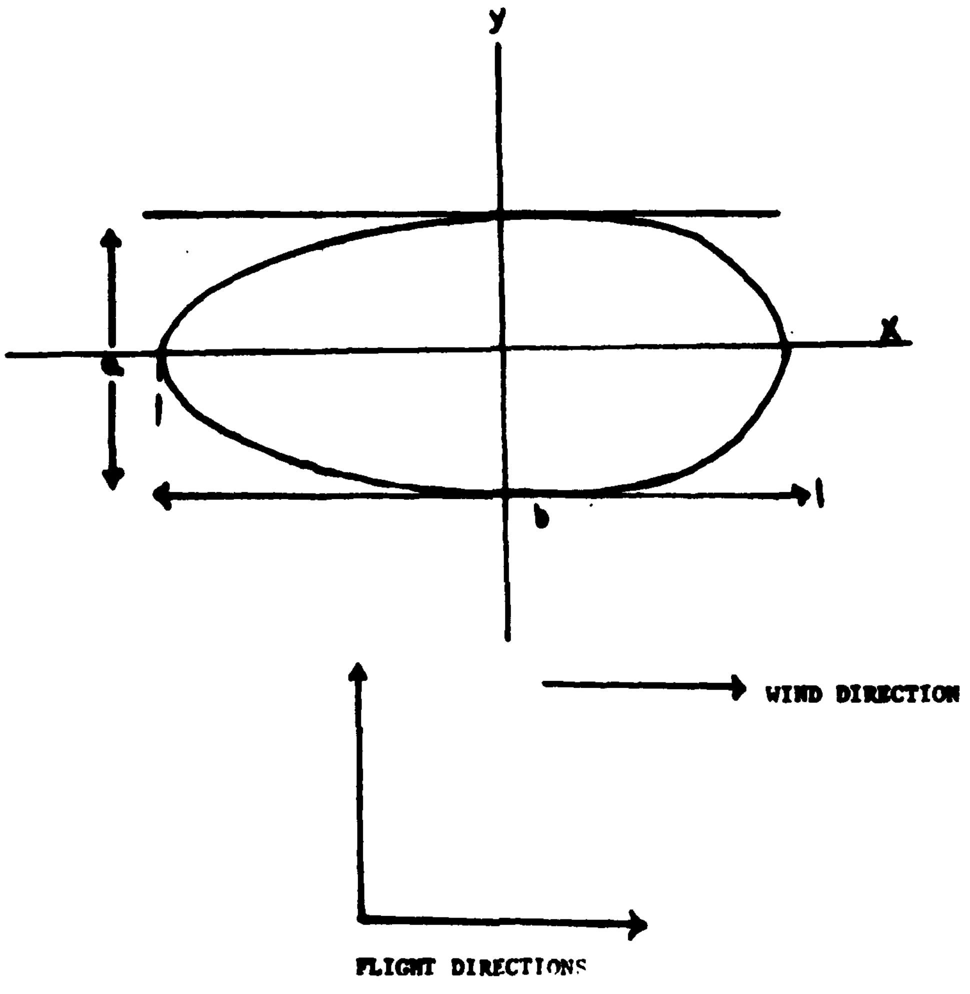


FIGURE 2

Model used for the horizontal cross-section of a convective element.

If they are ellipses with the same ratio of major to minor axis for all sizes of ellipses, then $\alpha_n = 0$.

Let $f^n_{2,n}(a_n)$ and $f^n_{1,n}(b_n)$ be the probablility density functions of the a and b axis of the horizontal elliptical cross sections. The probability density functions of the random lengths of the intersections with the convective elements in the x and y direction are related to $f^n_2(a_n)$, $f^n_1(b_n)$ by

$$w_{x,n}(l_{x,n}) = \frac{m_n^2 l_x}{\langle a \rangle} \int_{l'_{x,n}}^{a^*} \frac{(1+\alpha_n a_n)^2 f^n_2(a_n) da_n}{\sqrt{a_n^2 - m_n(1+\alpha_n a_n)^2 l^2_{x,n}}} \tag{3}$$

$$w_{y,n}(l_{y,n}) = \frac{l_{y,n}}{m_n^2 \langle b_n \rangle} \int_{l'_{y,n}}^{b^*} \frac{(1-\alpha_n m_n b_n)\ f^n_1(b_n) db_n}{\sqrt{b_n^2 - \frac{(1-m_n b_n)^2 l^2_{y,n}}{m_n^2}}} \tag{4}$$

where $l_{x,n}$, $l_{y,n}$ are the length of the random intersections of the convective element in the x and y directions respectively, $w_{x,n}(l_{x,n})$, $w_{y,n}(l_{y,n})$ are the measured probability density functions,

$$f^n_2(a) \equiv 0 \text{ for } a > a^*_n$$

$$f^n_1(b) \equiv 0 \text{ for } b > b^*_n$$

$$\langle a_n \rangle = \int_0^{a^*_n} a_n f^n_2(a_n) da_n \tag{5}$$

$$\langle b_n \rangle = \int_0^{b^*_n} b_n f^n_1(b_n) db_n \tag{6}$$

$$l'_{x,n} \equiv \frac{m_n l_{x,n}}{1-\alpha_n m_n l_{x,n}}$$

$$l'_{y,n} \equiv \frac{l_{y,n}}{m_n(1+\alpha_n l_{y,n})}$$

and the index n corresponds to a temperature excess $\theta_n = \theta_r + n\Delta\theta$ which will be dropped for brevity for the rest of this discussion.

Higher moments of l_x and l_y are given by

$$<l_x^p> = \int_0^\infty l_x^p w_x(l_x) dl_x \tag{7}$$

$$<l_y^p> = \int_0^\infty l_y^p w_y(l_y) dl_y \tag{8}$$

These can be shown to be

$$<l_x^p> = \frac{1}{m<a>} <\frac{a^{p+1}}{(1+\alpha a)^p}> B\ (\frac{p}{2} + \frac{1}{2}, \frac{1}{2}) \tag{9}$$

$$<l_y^p> = \frac{m^p}{<b>} <\frac{b^{p+1}}{(1-\alpha mb)^p}> B\ (\frac{p}{2} + \frac{1}{2}, \frac{1}{2}) \tag{10}$$

where $B(\frac{p}{2} + \frac{1}{2}, \frac{1}{2})$ is the beta function. Equation (7) can be rewritten as

$$<l_x^p> = \frac{m^p}{<a>} <\frac{b^{p+1}}{(1-\alpha mb)}> B\ (\frac{p}{2} + \frac{1}{2}, \frac{1}{2}) \tag{11}$$

Letting p = 1, and dividing equation (11) by (10) results in

$$\frac{<a>}{<b>} = \frac{<l_y>}{<l_x>} \tag{12}$$

and since

$$b = \frac{a}{m(1+\alpha a)}$$

$$\frac{a}{<\frac{a}{m(1+\alpha a)}>} = \frac{l_y}{l_x}$$

if $\alpha a^* < 1$, then

$$m^* = \frac{\langle l_y \rangle}{\langle l_x \rangle} \tag{13}$$

This value of m* will be used as a starting value to find α and the true m by a process of iteration using eqs. (3) and (9). The proper choice of m and α should give the proper computed values of $\langle l_x \rangle, \langle l_x^2 \rangle$ using eq. (9) and this result may be compared to the experimentally determined $\langle l_x \rangle$ and $\langle l_x^2 \rangle$.

Similarly the values of $\langle l_y \rangle$ and $\langle l_y^2 \rangle$ may be computed using eqs. (4) and (10), and compared with the experimentally determined $\langle l_y \rangle$, $\langle l_y^2 \rangle$.

The total area occupied by these ellipses with temperature excess of θ_n can be computed from the experimentally determined function of $f_2(a)$, and the parameters m and α.

The area of an ellipse is πab, so the average area of one ellipse is

$$\pi \langle ab \rangle = \frac{\pi}{m} \int_0^a \frac{a^2 f_2(a)}{(1+\alpha a)}\, da \tag{14}$$

which corresponds to a convective element with temperature excess of θ_n.

The number of intersections, n_x, of the aircraft with the convective elements in the x direction is given by

$$n_x = N_o \langle a \rangle L_x \tag{15}$$

where N_o is the number of ellipses per unit area and L_x is the distance traveled by the aircraft in the x direction. Thus the number of elements per unit area is

$$N_o = \frac{n_x}{\langle a \rangle L_x} = \frac{n_x}{L_x \int_0^{a^*} a f_2(a)\, da} \tag{16}$$

and the total area occupied by these elements per unit area with temperatures excess θ_n, is

$$\langle A\rangle = \pi N_o \langle ab\rangle = \frac{\pi n_x \int_o^{a^*} \frac{a^2 f_2(a)}{(1+\alpha a)}\, da}{m L_x \int_o^{a^*} a f_2(a)\, da} \tag{17}$$

IV. Stability of a Concentric Two-Cellular Atmospheric Vortex with Respect to Small Disturbances

by

K. H. Bergman

A study of the stability of a model vortex is currently being carried out, and final results of the study should soon be available. The particular vortex model chosen for study is a concentric two-cellular vortex with central downdraft and horizontal divergence within an "eye" structure, but horizontal convergence and ascent of air outside of the central "eye". Surface friction layer effects are not considered, and the horizontal flow components are assumed to be unchanging with height. This vortex model is essentially the one obtained by R. D. Sullivan (1959), as modified by H.-L. Juo (1966) for an unstable atmospheric layer.

An argument, based on those of H.-L. Kuo (1959, 1966) for concentric two-cellular structure of hurricane and tornado vortices, is applied to smaller scale atmospheric vortices such as dust-devils, and implies that, for the values of kinetic energy dissipation to be expected in a dust-devil vortex, a two-cellular structure should normally result when the vortex circulation is sufficiently well developed.

The equations of motion for the above vortex model are linearized for small disturbances, and the resulting set of differential equations is compared with those of Howard and Gupta (1962) for the case where viscous energy dissipation is neglected. Based on a criterion developed by Howard and Gupta, a sufficient condition for stability of axisymmetric disturbances is that

$$\beta^2 > \frac{r^3 (dW/dr)^2}{4d(rV)^2/dr}$$

where β is a dimensionless parameter defined by

$$\beta \equiv \frac{(rV)_\infty}{r_0 W_\infty}$$

and where r is the dimensionless radius, r_0 the radius of the vortex "eye", V and W the tangential and axial components of motion respectively for the undisturbed vortex, and the subscript (∞) refers to environmental conditions at a radius that is large compared to that of the vortex "eye".

The above criterion implies that the stable formation of a concentric two-cellular vortex depends upon a critical ratio of environmental angular momentum supply to the environmental updraft within which the vortex forms. Increased rotation of the vortex acts as a stabilizing influence, whereas increased vertical motion, because of the large horizontal shear of vertical motion developed in the vicinity of the "eye" boundary, tends to destabilize the vortex. This accords well with the observation that persistent organized vortex motion on a small scale is of comparatively infrequent occurrence in the atmosphere, compared to the frequency of buoyant plumes without organized circulation in which some, but apparently insufficient, net angular momentum supply can be expected.

Further study is being concentrated on establishing more precise stability criteria for the concentric two-cellular vortex model. The critical value of β for this model, based upon the Howard-Gupta criterion, is that $\beta \gtrsim 14.97$ is sufficient for vortex stability. However, this is a "weak" criterion which does not establish instability of the vortex for all values of β less than the above value, at least not for all disturbances. Hence the linearized equations have been expressed in finite-difference form and are being solved using an IBM 7094

computer in order to establish if, and for what values of β, disturbance wave number k, and axial Reynolds number r_0W_∞/v, solutions representing unstable disturbance modes exist.

Considerable difficulty in terms of computational stability has been encountered in solving the equations by numerical means, but preliminary results indicate tentatively that there is a critical value of wave number k above which all disturbances are stable. Inclusion of the viscous terms shows only a damping influence on the growth rate of unstable modes in the solutions obtained thus far.

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